DISCOVER

Science, Technology, and The Future

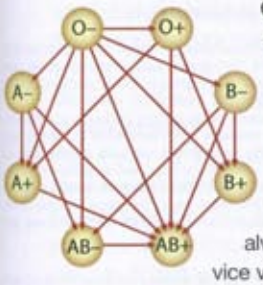
By Scott Kim

MATCHMAKER

"Matchmaker, matchmaker, make me a match"
—Fiddler on the Roof

Optimized Match (optimizedmatch.com) is a technique developed by mathematician Sommer Gentry and transplant surgeon Dorry Segev to increase the odds that a patient in need of an organ transplant will find a suitable donor. It is based on graph theory, a branch of mathematics used to analyze diagrams made of dots connected by lines. Here are some puzzles inspired by graph theory.

BLOOD TYPES



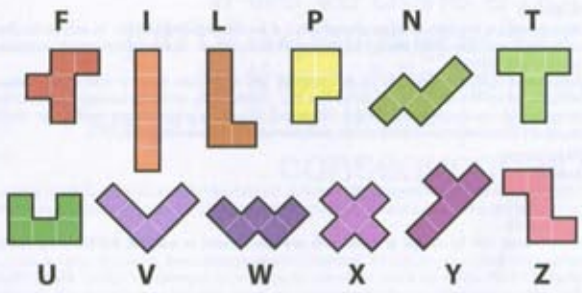
Graph theory is used to model things like the structure of molecules, the strength of buildings, and the connectivity of the Internet. Here is a medical application. The diagram at left shows the eight major blood types. An arrow from one to another, say A- to A+, indicates that the first can donate to the second. Notice that compatibility is not always mutual: A- can donate to A+, but not vice versa. (All blood is compatible with itself, but here we are interested only in donations between different types.)

1. [Easy] Which type can donate to the largest number of other blood types? Which type can receive the largest number of other types of blood?

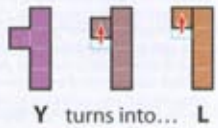
2. [Medium] How many ways are there to make a chain of four blood types so that the first can donate to the second, the second to the third, and the third to the fourth? For instance, one solution is O- to B- to B+ to AB+.

3. [Tricky] Make a group of three blood types, none of which can donate to either of the other two. Can you find both possible solutions?

SHAPE COUSINS

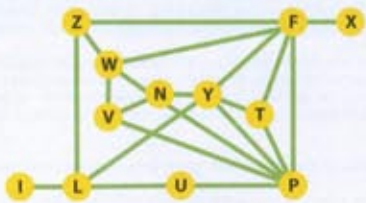


Here is a puzzle that looks very different from organ donation but uses the same underlying mathematics. The 12 shapes above are each made of five squares. For convenience, I've labeled them with letters that look like the shapes. Suppose I tell you that two shapes are "cousins" if you can change one shape into the other by sliding one of the squares one space horizontally, vertically, or diagonally. (After sliding one square, you may also rotate or flip the whole shape.) For instance, Y is a cousin of L and T, but not of X.



1. [Easy] Y is a cousin of three other shapes. What are they?

2. [Medium] Which shapes have only one cousin?



3. [Tricky] Can you divide the 12 shapes into six pairs of cousins? Hint: Study the diagram at left.

ORGAN SWAP

A problem in organ transplants is that someone who wants to donate an organ to a loved one often can't because their blood types are incompatible. The solution is to match such a couple with another couple who have the right blood types, and swap organs, the donor from each couple giving a kidney to the patient in the other couple. The operations must take place simultaneously and at the same facility. Johns Hopkins and several other institutions have performed over a hundred such procedures, a few of which have involved swaps among more than two couples. Gentry and Segev now advocate a nationwide registry to spread the practice.

In the diagram below, each circle represents two people: a patient who needs a kidney and a donor who wants to donate a kidney to that patient. The people in each couple have incompatible blood types, so the donor cannot give a kidney to his or her partner. Each line between two couples represents a potential swap: The donor from each pair is compatible with the patient in the other couple.

1. [Easy] Which couples have only one potential match?

2. [Tricky] Optimized Match finds the best swapping scheme for a pool of available donors. Maximize the number of matches by choosing seven lines that connect all 14 couples. Hint: The solution is unique.

3. [Tricky] If you do not choose swaps carefully, you may end up stranding some couples. Find the worst possible matching scheme such that no more than five lines can be chosen. You are not allowed to choose two lines that end at the same couple. Hint: The solution is not unique.

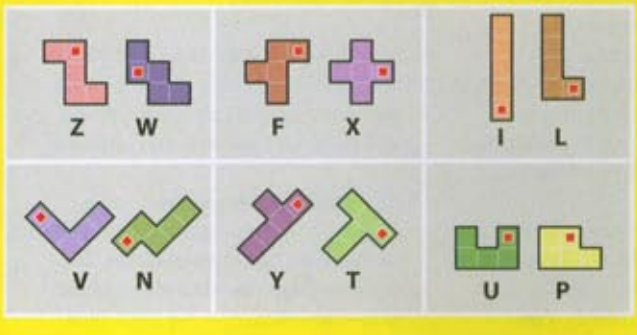
ANSWERS ON PAGE 79

MIND GAMES
SOLUTIONS

SHAPE COUSINS

1. Y is also a cousin of N, F, and P.

2. There are two shapes that have only one cousin each: I is a cousin only of L, and X is a cousin only of F.

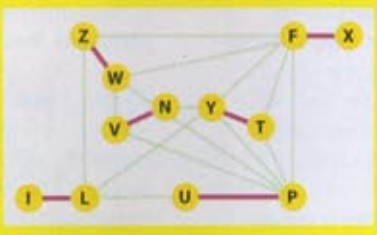


3. Above is the unique solution. The square that changes one shape to its cousin is marked with a small red square.

You can find the solution by exhaustively trying all combinations, but it is easier to first diagram all the relationships (graph theorists call such a diagram a graph).

In the graph below, each shape is represented by a node, which is connected to each of its cousins by a line. Dividing the 12 shapes into six pairs of cousins is the same as finding six lines in the graph whose endpoints include all 12 nodes. Notice that no two of these lines will end at the same node, since no shape can appear in two different pairs.

Start by choosing the lines connecting I to L and F to X, since these choices are forced (I connects only to L, and F connects only to X). Remove all the remaining lines connected to L and F. Nodes U and Z now have only one cousin each, which forces you to pair them with P and W. Finally, the remaining nodes V, N, Y, and T must be paired as V-N and Y-T. The result is a graph that contains only the six red lines.



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BLOOD TYPES

1. Type O- can donate to all the other blood types. Type AB+ can receive from all the other blood types.

2. There are six possible such chains:
O- to O+ to A+ to AB+
O- to O+ to B+ to AB+
O- to A- to A+ to AB+
O- to A- to AB- to AB+
O- to B- to B+ to AB+
O- to B- to AB- to AB+

3. There are two such sets of three mutually incompatible blood types: (A-, B-, O+) and (A+, B+, AB-).

ORGAN SWAP

1. Couples 1 and 7 have only one potential match because there is only one one line ending at each of these circles.

2.

3.